Test cases were also run for problems in which the integral term in Eq. (1) included penalties involving products of x and u. Such penalty terms were found to be necessary for the states to converge in some control smoothing problems.

Conclusions

A simple iterative algorithm has been presented for selecting terminal penalty weighting matrices for linear-quadratic control problems. The algorithms selects a weighting matrix which yields acceptable system performance, while avoiding the large changes in magnitude in final values of the feedback gains. The algorithm is made computationally efficient by using closed-form solutions for the time-varying Riccati equation and the closed-loop system response. The method is heuristic, but offers a systematic approach to terminal weight selection.

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Comparison of Angular and Metric Guidance Laws for Tactical Missiles

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Introduction

IN 1979, Nesline and Zarchan[1] compared classical and modern homing guidance for tactical missiles. The classical, or proportional navigation (PN), laws correspond to the application of a filter to the missile-to-target line-of-sight (LOS) rotation rate. Guidance based on such laws can be termed "angular." A useful analytical tool in this context is the adjoint technique,[2,3] first introduced by Bennet and Mathews[4] and used by Nesline and Zarchan[5] to determine the performance of a given guidance loop. However, there is nothing in the open literature concerning the use of this technique for the synthesis of an optimal filter, which is possible.

The laws describing modern guidance systems are based on the methods of optimal control, which have been suitably summarized by Bryson and Ho.[6] Invoking the separation theorem, these laws are presented as comprising an estimator and a deterministic controller in cascade. These laws can be termed "metric" if they process the Cartesian coordinates of the missile-to-target vector. The miss distance can be minimized using many possible terminal control criteria,[7] or its estimate can be forced to zero.[8]

This Note compares an angular guidance law optimized by the adjoint technique and a metric law comprising an optimal estimator connected to a terminal controller forcing the estimated miss distance to zero. The terms of comparison are miss distance and realizability. An analytical solution is given for one specific but significant case. To the author's knowledge, this is the first time such a solution has been published.

Optimal Metric Law

The guidance model is unidimensional and linearized about the mean collision line. It is assumed that the time to go, $t_{go}=t_f-t$, and the relative missile-to-target velocity, V_c ($V_c>0$), are measured. The corresponding equations are:

$$\ddot{y}=a_T, \qquad \ddot{y}_M=a_M \tag{1}$$

$$y_M-y_T=-\sigma V_c t_{go}+n \tag{2}$$

where y_M and y_T are the missile and target coordinates relative to the mean collision line, σ the angle between the LOS and the mean collision line, a_M and a_T the missile and target lateral accelerations, and n denotes the zero-mean metric noise. Further,

$$a_M=P(s)a_c \tag{3}$$

where a_c is the missile commanded lateral acceleration and $P(s)$ the autopilot transfer function.

The optimal control law comprises an optimal state estimator dependent upon measurement noise n and process noise a_T, connected to a terminal controller which, in the absence of noise, forces the miss distance $y_{TM}(t_f)=y_M(t_f)-y_T(t_f)$ to zero.

$$a_c(t)=C(t)\hat{y}(t) \tag{4}$$

$\hat{y}(t)$ denotes the estimate of the state vector $y(t)$. The components of this vector are $y_{TM}(t)$, $\dot{y}_{TM}(t)$, and other variables depending on the manner in which a_T is modeled. If no limitations are placed on lateral acceleration, then the miss distance, σ_d, depends solely on measurement noise and target maneuvering (TM).

$$\sigma_d^2=E[\tilde{y}_{TM}^2(t_f,n,a_T)] \tag{5}$$

Assuming that measurement noise and target maneuvering are stationary with spectral densities $\phi_{nn}(s)=N(s)N(-s)$ and $\phi_{TT}(s)=T(s)T(-s)$, then the asymptotic estimator minimizing $\sigma_d^2=E[\tilde{y}_{TM}^2(\infty)]$ is a Wiener filter with transfer function $W(s)$, as indicated by the diagram illustrated in Fig. 1. The corresponding equation is

$$\sigma_d^2=\frac{1}{2\pi j}\int_{-j\infty}^{j\infty}\left[\phi_{TT}(s)\left(\frac{1-W(s)}{s^2}\right)\left(\frac{1-W(-s)}{s^2}\right)+\phi_{nn}(s)W(s)W(-s)\right]\mathrm{d}s \tag{6}$$

The quantity a_M is assumed to be known either by direct measurement or by simulating $P(s)$. This yields $\tilde{y}_{TM}=\tilde{y}_T$.

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Figure 1 can also be interpreted deterministically. $d = \tilde{y}_{TM}(t_f)$ is the miss distance caused by a target maneuver $a_T = \mathcal{L}^{-1}\, T(s)$ beginning at time $t = 0$. Thus the miss distance $d(t_{go})$, where t_{go} is the time to go at which the maneuver is initiated, is given by

$$d(t_{go}) = \mathcal{L}^{-1}\left(\frac{T(s)}{s^2}(1 - W(s))\right) \tag{7}$$

Note that the miss distance is independent of the time to go at which the controller described by Eq. (4) is activated, it being understood that the estimation begins, in all cases, at $t_{go} = \infty$. On the other hand, the variation of both missile acceleration and guidance error $y_{TM}(t)$, as a function of time, depends, of course, on the time at which the controller is activated.

Optimal Angular Guidance

The classical linear angular guidance loop is illustrated in block diagram form in Fig. 2. $S(s)$ is the seeker transfer function and $C(s)$ the guidance filter to be optimized.

If $M(s)$ is the overall missile transfer function, we can write $M(s) = S(s)C(s)P(s)$. Further, if $x(t_{go})$ is the impulse response of the adjoint system and $X(s)$ its Laplace transform, as indicated in Fig. 3, then, as is known,[2,3,5] $X(s)$ can be used to compute the miss distance induced by measurement noise $\phi_{nn}(s)$, stochastic target maneuver $\phi_{TT}(s)$, and by a deterministic target maneuver beginning at time to go t_{go}, having the function $T(s)$ as its Laplace transform.

$$\sigma^2_{d_n} = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} (X(s) - 1)(X(-s) - 1)\phi_{nn}(s)\mathrm{d}s \tag{8}$$

$$\sigma^2_{d_T} = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} \left(\frac{X(s)}{s^2}\right)\left(\frac{X(-s)}{s^2}\right)\phi_{TT}(s)\mathrm{d}s \tag{9}$$

$$d(t_{go}) = \mathcal{L}^{-1}\frac{X(s)}{s^2}T(s) \tag{10}$$

Equations (8) and (9) allow the computation of $X_0(s)$ minimizing $\sigma^2_d = \sigma^2_{d_T} + \sigma^2_{d_n}$, the total miss distance.

$$\sigma^2_d = \frac{1}{2\pi j}\int_{-j\infty}^{j\infty} \left((X_0(s) - 1)(X_0(-s) - 1)\phi_{nn}(s) + \left(\frac{X_0(s)}{s^2}\right)\left(\frac{X_0(-s)}{s}\right)\phi_{TT}(s)\mathrm{d}s\right. \tag{11}$$

Comparison of Optimal Angular and Optimal Metric Laws

Comparison of Eqs. (11) and (6) shows that the minimal miss distance variances are equal and that the following relation holds.

$$W(s) = 1 - X_0(s) \tag{12}$$

Moreover, application of Eqs. (7) and (10) shows that the miss distances induced by a deterministic target are also equal.

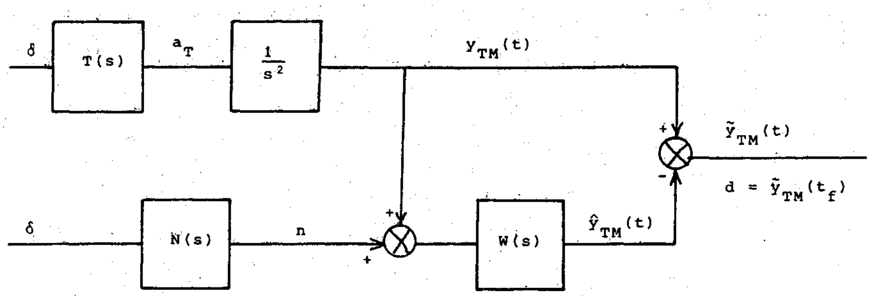

Fig. 1 Wiener filter as metric estimator.

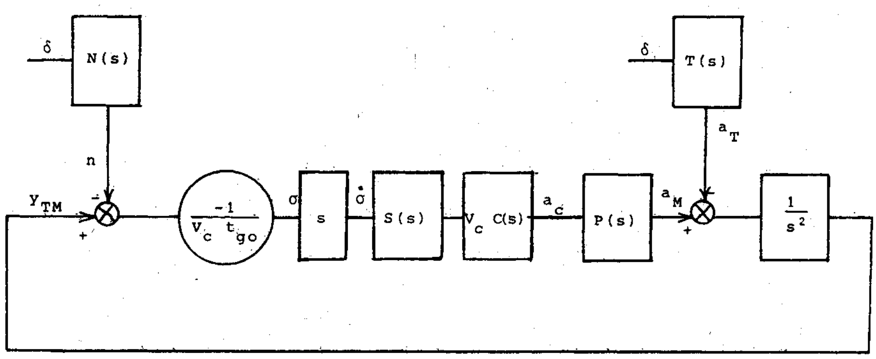

Fig. 2 Angular guidance loop.

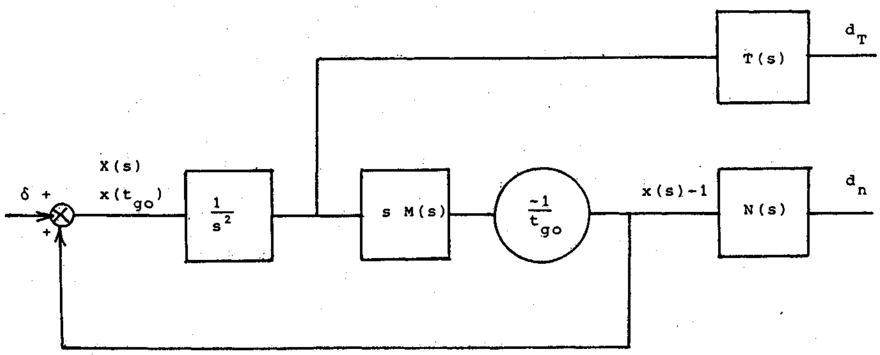

Fig. 3 Adjoint angular guidance loop.

Of course the missile acceleration and guidance error histories will be different in the two cases, if only because the metric law gives an indefinite number of histories from the time to go at which the controller is activated.

In the relatively general case[9,10] where $\phi_{nn}(s) = k_n^2$ (white metric noise) and $\phi_{TT}(s) = k_{T_p}^2/s^{2p}$, the Wiener-Hopf equation giving $W(s)$ and $X_0(s)$ has an analytical solution. The poles of $W(s)$, $X_0(s)$, and $M_0(s)$ follow a Butterworth configuration of degree $p+2$ with $\omega = (k_{T_p}/k_n)^{1/(p+2)}$. $M_0(s)$ is given by

$$M_0(s) = \frac{m_1 s^{p+1} + 2m_2 s^{p+2} + \ldots + (p+1)m_{p+1}s + (p+2)\omega^{p+2}}{s^{p+2} + m_1 s^{p+1} + \ldots + m_{p+1}s + \omega^{p+2}} \tag{13}$$

where m_i are the coefficients of the Butterworth polynomial. The miss distance is $\sigma_a^2 = m_1 k_n^2$. In the angular case, the proportional navigation ratio is $p+2$.

Clearly, in most cases, the optimal angular guidance filter $C_0(s) = M_0(s)S^{-1}(s)P^{-1}(s)$ is not realizable. In contrast, the optimal metric law is always realizable.

Conclusion

Linear-Quadratic-Gaussian optimization, without constraint on commanded acceleration and with stationary measurement and process noise, yields the same theoretical minimum miss distance irrespective of whether the control law is angular or metric. Generally, however, the optimal angular law cannot be realized exactly. If an approximate angular law such as simple proportional navigation is used, the resulting performance degradation is inversely proportional to measurement noise. The metric law produces a performance advantage as a result of time-to-go measurement, which produces a better missile lateral acceleration history against a maneuvering target. As is known, the specific problems of radome design and seeker-to-missile coupling[1] are such that the implementation of an optimal control law is more effective in the case of command-to-line-of-sight guidance than with homing guidance.[10]

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On-Off Attitude Control of Flexible Satellites

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Introduction

THE reorientation of a satellite is often more rapidly achieved using thrusters rather than internal momentum transfer devices. However, thrusters have the disadvantages of being difficult to adjust with precision and of tending to excite elastic modes of vibration. Feedback control schemes[1] have been offered for thruster control based on pulse-width and pulse-frequency modulation. If, however, control moment gyros or other internal mechanisms are available to take over the fine control once the maneuver is complete, an open-loop approach for thruster-based attitude acquisition may be useful. Open-loop control will be especially attractive if the switching times can be selected in such a way as to effectively minimize the postmaneuver elastic energy of the system.

The present work is an examination of the relationship between post maneuver elastic energy and switch time selection. It is conducted in the context of a simple satellite model that has served as the basis for other flexible space structure control studies.[2,3] The control is restricted in that only three switching times can be selected: 1) time T_1, the duration of the first thrust interval; 2) time T_2 at which a braking interval begins; and 3) time T_3 at which the braking interval ends. A final time T_F, measured from the beginning of the first thrust interval, is specified before which the maneuver is to be completed.

Assuming that the system begins from rest, two requirements are placed upon the maneuver: 1) a prescribed angular impulse (resulting in the desired final system angular velocity) should be delivered, and 2) a prescribed time integral of the angular impulse is to be delivered at T_F. In the absence of an additional constraint or condition, an infinite number of switch time combinations T_1, T_2, and T_3 could be selected such that the maneuver is completed by time T_F. The selection of this third condition is shown to have a significant impact upon the final presence or absence of elastic energy.

The Model

Consider the rigid hub and flexible appendage model of Fig. 1. If the four cantilevered appendages are elastic, of constant cross section, and relatively long and thin, then the equations that govern the response of the angular position $\theta(t)$ of the rigid hub and that of the appendage deflection $y(z,t)$ to the control torque u are given by

$$I_T\left(\frac{d^2\theta}{dt^2}\right) + 4\rho\int_{R_i}^{R_o} z\left(\frac{\partial^2 y}{\partial t^2}\right)dz = u \tag{1}$$

$$EI\left(\frac{\partial^4 y}{\partial z^4}\right) + \rho\left[\left(\frac{\partial^2 y}{\partial t^2}\right) + z\left(\frac{d^2\theta}{dt^2}\right)\right] = 0 \tag{2}$$

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